String and Brane Tensions as Dynamical Degrees of Freedom

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Abstract

We discuss a new class of string and p-brane models where the string/brane tension appears as an *additional dynamical degree of freedom* instead of being introduced by hand as an *ad hoc* dimensionfull scale. The latter property turns out to have a significant impact on the string/brane dynamics. The dynamical tension obeys Maxwell (or Yang-Mills) equations of motion (in the string case) or their rank p gauge theory analogues (in the p-brane case), which in particular triggers a simple classical mechanism of ("color") charge confinement.

Keywords: modified string and p-brane actions, reparametrization-covariant integration measures, dynamical generation of string/brane tension, color charge confinement.

1 Introduction

In order to build actions describing dynamics in geometrically motivated field theories (for reviews of string and brane theories, see [1]) we need among other things a consistent generally-covariant integration measure density, *i.e.*, covariant under arbitrary diffeomorphisms (reparametrizations) on the underlying space-time manifold. Usually the natural choice is the standard Riemannian metric density $\sqrt{-g}$ with $g \equiv \det ||g_{\mu\nu}||$. However, there are no purely geometric reasons which prevent us from employing an alternative generally-covariant integration measure. For instance, introducing additional D scalar fields φ^i ($i = 1, \ldots, D$ where D is the space-time dimension) we may take the following new non-Riemannian measure density $\Phi(\varphi)$:

Using (**??**) allows to construct new classes of models involving Gravity called *Two-Measure Gravitational Models* [2], whose actions are typically of the form:

For a recent review of two-measure gravity models see the contribution in this volume [3]. In what follows we are going to apply the above ideas to the case of string and p-brane models. Part of our exposition is based on earlier works [4]. Furthermore, we will elaborate on various important properties of the modified-measure string and brane models with dynamical string/brane tension.

2 Bosonic Strings with a Modified World-Sheet Integration Measure

We begin by first recalling the standard Polyakov-type action for the bosonic string [5]:

Let us now introduce two additional world-sheet scalar fields φ^i $(i = 1, 2)$ and replace $\sqrt{-\gamma}$ with a new reparametrization-covariant world-sheet integration measure density $\Phi(\varphi)$ defined in terms of φ^i :

To remedy the above situation let us consider topological (total-derivative) terms w.r.t. standard Riemannian world-sheet integration measure. Upon measure replacement $\sqrt{-\gamma} \to \Phi(\varphi)$ the former are *not any more* topological – they will contribute nontrivially to the equations of motion. For instance:

Eq.(**??**) prompts us to construct the following consistent modified bosonic string action[1] :

The equations of motion w.r.t. φ^i resulting from (**??**) :

The equations of motion w.r.t. X^μ read:

Most importantly, the equations of motion w.r.t. A_a resulting from (**??**) yield:

Since the modified-measure string model (**??**) naturally requires the presence of the auxiliary Abelian world-sheet gauge field A_a, we may extend it by introducing a coupling of A_a to some world-sheet charge current j^a :

The rest of the Hamiltonian constraints are $\pi_{A_0} = 0$ and

3 Classical Confinement Mechanism of "Color" Charges via Dynamical String Tension

3.1 Non-Abelian Generalization

First, let us notice the following identity in $D = 2$ involving Abelian gauge field A_a:

The action (**??**) is again invariant under the Φ-*extended Weyl (conformal)* symmetry (**??**).

Notice that the *"square-root" Yang-Mills* action (with the regular Riemannian-metric integration measure):

Similarly to the Abelian case (**??**) we can also add a coupling of the auxiliary non-Abelian gauge field A_a to an external "color"-charge world-sheet current j^a:

The action (**??**) produces the following equations of motion w.r.t. φ^i and γ^{ab}, respectively:

[1] In ref.[6] another interesting geometric modification of the standard bosonic string model has been proposed, which is based on dynamical world-sheet metric and torsion.

The equations of motion w.r.t. auxiliary gauge field A_a resulting from (**??**) resemble, similarly to the Abelian case (**??**), the $D = 2$ non-Abelian Yang-Mills equations:

The equations of motion for the dynamical string tension following from (**??**) is:

Finally, the X^μ-equations of motion $\partial_a \left(\Phi(\varphi)\gamma^{ab}\partial_b X^\mu\right) = 0$ resulting from the action (**??**) can be rewritten in the conformal gauge $\sqrt{-\gamma}\gamma^{ab} = \eta^{ab}$ as:

For static charges $\widetilde{j}^0 = -\sum_i \widetilde{e}_i \delta(\sigma - \sigma_i)$:

3.2 Classical Confinement Mechanism

Recall that the modified string action (**??**) yields the $D = 2$ Yang-Mills-like Eqs.(**??**) whose 0-th component $\partial_\sigma \mathcal{E} + i\Big[A_1\,,\,\mathcal{E}\Big] + j^0 = 0$ is the "Gauss law" constraint for the dynamical string tension ($T \equiv |\mathcal{E}| = \Phi(\varphi)/\sqrt{-\gamma}$). For point-like "color" charges and taking the gauge $A_1 = 0$ (*i.e.*, $\mathcal{E} \to \widetilde{\mathcal{E}} = G\mathcal{E}G^{-1}$ where $A_1 = -iG^{-1}\partial_\sigma G$), the latter reads:

Let us consider the case of *closed* modified string with positions of the "color" charges at $0 < \sigma_1 < \ldots < \sigma_N \leq 2\pi$. Then, integrating the "Gauss law" constraint (**??**) along the string (at fixed proper time) we obtain:

The discussion in this section leads to the following conclusions:

- We see from Eqs.(**??**)–(**??**) that the modified-measure (closed) string with N point-like ("color") charges on it ((**??**) or (**??**)) is equivalent to N chain-wise connected regular open string segments obeying Neumann boundary conditions.
- Each of the above open string segments, with end-points at the charges e_i and e_{i+1} (in the Abelian case) or C_i and C_{i+1} (in the non-Abelian case), has *different* constant string tension $T_{i,i+1}$ such that $T_{i,i+1} = T_{i-1,i} + \overset{(\sim)}{e}_i$ (the non-Abelian $\widetilde{e}_i$ are defined in (**??**)).
- Eq.(**??**) tells us that the only (classically) admissable configuration of "color" point-like charges coupled to a modified-measure closed bosonic string is the one with *zero* total "color" charge, *i.e.*, the model (**??**) provides a classical mechanism of "color" charge confinement.

4 Branes with a Modified World-Volume Integration Measure

Before generalizing our construction from the previous two sections to the case of higher-dimensional p-branes, let us recall the standard Polyakov-type formulation of the bosonic p-brane action:

4.1 Modified-Measure Brane Actions

Now, similarly to the string case we introduce a modified world-volume integration measure in terms of $p + 1$ auxiliary scalar fields φ^i $(i = 1, \ldots, p + 1)$:

The requirement for $\Omega(A)$ to be a topological density is dictated by the requirement that the modified-measure brane action (**??**) (in the absence of the last gauge/matter term $\int d^{p+1}\sigma\, \mathcal{L}(A)$) reproduces the ordinary p-brane equations of motion apart from the fact that the brane tension $T \equiv \Phi(\varphi)/\sqrt{-\gamma}$ is now an *additional dynamical degree of freedom*.

The simplest example of a topological density $\Omega(A)$ for the auxiliary gauge/matter fields is:

More generally, for $p+1 = rs$ we can have:

We may also employ *non-Abelian* auxiliary gauge fields as in the string case. For instance, when $p = 3$ we may take:

The modified p-brane action (**??**) produces the following equations of motion w.r.t. φ^i :

We now consider the modified brane (**??**) equations of motion w.r.t. auxiliary (gauge) fields A^I – these are the eqs. determining the dynamical brane tension $T \equiv \Phi(\varphi)/\sqrt{-\gamma}$:

As a physically interesting example let us take the choice (**??**) for the topological density $\Omega(A)$ and consider the following natural coupling of the auxiliary p-form gauge field:

Finally, the modified-brane action (**??**) yields the X^μ-equations of motion $\partial_a \left(\Phi(\varphi)\gamma^{ab}\partial_b X^\mu\right) = 0$ (taking for simplicity $G_{\mu\nu} = \eta_{\mu\nu}$), which upon using (**??**) can be rewritten in the form [2] :

4.2 Confinement of Charged Lower-Dimensional Branes

Let us consider the solutions for the for the dynamical brane tension Eq.(**??**). Recalling the definition (**??**) of $\mathcal{N}_a^{(i)}$ we find from (**??**) that $T \equiv \Phi(\varphi)/\sqrt{-\gamma}$ is piece-wise constant on the p-brane world-volume with jumps when crossing the world-hypersurface of each charged $(p-1)$-sub-brane $\mathcal{B}_i$, the corresponding jump being equal to the charge magnitude $\pm e_i$ (the overall sign depending on the direction of crossing w.r.t. the normal $\mathcal{N}_a^{(i)}$).

Taking into account the above piece-wise constant solution for $T \equiv \Phi(\varphi)/\sqrt{-\gamma}$, the X^μ-equations of motion (**??**) for the closed modified brane (**??**) become equivalent to the following set of equations of motion:

Integrating Eq.(**??**) along arbitrary smooth closed curve $\mathcal{C}$ on the p-brane world-volume which is transversal to (some or all of) the $(p-1)$-sub-brane $\mathcal{B}_i$, we obtain the following constraints on the possible sub-brane configurations:

As a simple illustration, here we will only consider the simplest non-trivial case $p = 2$ and take the static gauge for the $p = 1$ sub-branes (strings), *i.e.*, the proper times of the charged strings coinsides with the proper time of the bulk membrane. The latter means that the fixed-time world-volume of the bulk *closed* membrane is a Riemann surface with some number g of handles and no holes. Further, we will assume the following simple topology of the attached N charged strings $\mathcal{B}_i$: upon cutting the

[2] For a detailed description of techniques for obtaining solutions of equations of motion for standard string and brane systems in non-trivial backgrounds, which can be easily adapted in the present modified-measure string and brane models, see ref.[8].

membrane surface along these attached strings it splits into N *open* membranes $\mathcal{M}_i$ ($i = 1, \ldots, N$) with Neumann boundary conditions (cf. (**??**)), each of which being a Riemann surface with g_i handles and 2 holes (boundaries) formed by the strings $\mathcal{B}_{i-1}$ and $\mathcal{B}_i$, respectively[3]. The brane tension of $\mathcal{M}_i$ is a dynamically generated constant T_i where $T_{i+1} = T_i + e_i$. In the present configuration Eq.(**??**) evidently reduces to the constraint $\sum_i e_i = 0$.

Thus, we conclude that similarly to the string case, modified-measure p-brane models describe configurations of charged $(p-1)$-branes with charge confinement. Apart from the latter, in general there exist more complicated configurations allowed by the constraint (**??**), which will be studied elsewhere.

5 Conclusions

The above discussion shows that there exist natural from physical point of view modifications of world-sheet and world-volume integration measures which may significantly affect string and brane dynamics. Let us sumarize the main features of the new modified-measure string and brane models:

- Acceptable dynamics *naturally* requires the introduction of auxiliary world-sheet gauge field (world-volume p-form tensor gauge field).
- The string/brane tension is *not* anymore a constant scale given *ad hoc*, but rather appears as an *additional dynamical degree of freedom* beyond the ordinary string/brane degrees of freedom.
- The dynamical string/brane tension has physical meaning of an electric field strenght for the auxiliary gauge field.
- The dynamical string/brane tension obeys “Gauss law” constraint equation and may be non-trivially variable in the presence of point-like charges (on the string world-sheet) or charged lower-dimensional branes (on the p-brane world-volume).
- Modified-measure string/brane models provide simple classical mechanisms for confinement of point-like “color” charges or charged lower-dimensional branes due to variable dynamical tension.

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[3]The Euler charasteristics of the bulk membrane Riemann surface is $\chi = 2 - 2g$, whereas for the open brane $\mathcal{M}_i$ it is $\chi_i = 2 - 2g_i - 2$, so that $\chi = \sum_i \chi_i$ or, equivalently, $g = 1 + \sum_i g_i$.